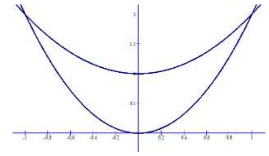
SOLUTION

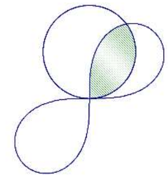
Question 1 (2pt). Intersections $x = -1 \vee x = 1$



a. $\int_{-1}^{1}(x^2+1-2x^2)dx = 4/3$

b. $\int_{-1}^{1}(x^2+1-2x^2)^2dx = \frac{16}{15}$

Question 2 (1pt).



$$\int_0^{\pi/4}\frac{1}{2}(2\sin(2\theta))^2d\theta + \int_{\pi/4}^{\pi/2}\frac{1}{2}(4\sin(2\theta))d\theta = \frac{\pi}{4}+1$$

Question 3 (1 pt).

$$I = \lim_{N\to\infty}\int_2^N \frac{1}{x(x^2+4)}dx = \lim_{N\to\infty}\frac{1}{8}\ln\left(\frac{x^2}{x^2+4}\right)\Bigg|_2^N = \lim_{N\to\infty}\frac{1}{8}\left(\ln\left(\frac{N^2}{N^2+4}\right)-\ln\frac{1}{2}\right) = \frac{\ln 2}{8}$$

Question 4 (1.5 pt). $y' - y = xe^x, \quad y(0) = 1.$

- $I(x) = e^{-x}$.
- $y(x) = e^x\left(\frac{x^2}{2}+C\right)$;
- $C = 1$
- $y(x) = e^x\left(\frac{x^2}{2}+1\right)$;

Question 5 (1pt).

$$3.4\overline{25} = 3.4 + 25.10^{-3} + 25.10^{-5} + \cdots$$
$$= 3.4 + \sum_{k=0}^{+\infty}(25\times 10^{-3})(10^{-2})^k\,;$$
$$3.4\overline{25} = \frac{34}{10} + \frac{25\times10^{-3}}{1-10^{-2}} = \frac{34}{10}+\frac{25}{990} = \frac{3391}{990}.$$

Problem 6 (1 pt).

$$0 \le \frac{5-\cos\pi k}{3k^2+\ln k} \le \frac{6}{3k^2+\ln k} \le \frac{6}{3k^2} = \frac{2}{k^2}, \forall k \ge 1$$

Since $\sum_{k=1}^{\infty}\frac{2}{k^2}$ converges ($p = 2 > 1$) then the given series converges by the direct comparison test.

Question 7 (1.5 pts).

- $L = 2|x-3|$.
- $x = \frac{5}{2}$, the series converges by Alternating series test.
- $x = 7/2$, the series diverses by limits comparision test
- The interval of convergence is [5/2,7/2)

Question 8 (1pt).

- $\overrightarrow{CH} = \langle 1, \frac{3}{2} - m\rangle$, $\overrightarrow{AB} = \langle 2, -1\rangle$.
- $\overrightarrow{CH} \perp \overrightarrow{AB} \Leftrightarrow 2 - \left(\frac{3}{2} - m\right) = 0 \Leftrightarrow \frac{1}{2} + m = 0 \Leftrightarrow m = \frac{-1}{2}$
- With $m = -\frac{1}{2}$, the point $C\left(1, \frac{-1}{2}\right)$ and $AB = \sqrt{2^2+(-1)^2} = \sqrt{5}$, $CH = \sqrt{1^2+2^2} = \sqrt{5}$.
- The area of the triangle ABC is $A = \frac{1}{2}AB.CH = \frac{5}{2}$.