

**HCMC UNIVERSITY OF TECHNOLOGY AND EDUCATION**  
**HIGH QUALITY TRAINING FACULTY**  
**SOLUTION OF CALCULUS 2- HK2 2022-2023**

Content	Score
<p><b>1</b></p> $\text{Area} = \frac{1}{2} \int_0^{2\pi} (5 - 3\cos\theta)^2 d\theta.$ $= \frac{59\pi}{2}.$	0.5  0.5
<p><b>2</b></p> $\int_2^\infty \frac{t}{(t^2 - 1)^3} dt = \lim_{N \rightarrow \infty} \left( \int_2^N \frac{t}{(t^2 - 1)^3} dt \right)$ <p>Let <math>u = t^2 - 1</math>. Then <math>\lim_{N \rightarrow \infty} \left( \int_2^N \frac{t}{(t^2 - 1)^3} dt \right) = \lim_{N \rightarrow \infty} \left( \int_1^{N^2 - 1} \frac{du}{2u^3} \right)</math></p> $= \lim_{N \rightarrow \infty} \left( \frac{1}{4} - \frac{1}{4(N^2 - 1)^2} \right)$ $= 1/4$	0.25  0.5  0.25
<p><b>3</b></p> $\text{Volume} = 2\pi \int_0^4 (x + 1)(4x - x^2) dx.$ $= 64\pi$	0.5  0.5
<p><b>4</b></p> <p>Root test: <math>\lim_{k \rightarrow \infty} (k - 2 / k)^k = e^{-2} \neq 0</math></p> <p>The series diverges.</p>	0.5  0.25  0.25
<p><b>5</b></p> $\sum_{k=0}^{\infty} \frac{2023^k}{10^{4k-1}} = 10 \left( \sum_{k=0}^{\infty} \left( \frac{2023}{10000} \right)^k \right)$ $= \frac{10}{1 - 2023 / 10000}$	0.5  0.5
<p><b>6</b></p> $\lim_{k \rightarrow \infty} \sqrt[k]{\left  \frac{(k^2 + 20k)}{23^k} (x + 3)^k \right } = \frac{ x + 3 }{23}$	0.5

	The radius of convergence is $1/23$ If $x+3=23$ or $x+3=-23$ then the series diverges	0.5 0.5
7	The Maclurin series for $\sin(2x)$ is $2x - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} - \dots$ The Maclurin series for $x\sin(2x)$ is $2x^2 - \frac{(2x)^3 x}{3!} + \frac{(2x)^5 x}{5!} - \dots$ coefficient of $x^4$ is: $8/6$	0.5 0.5
8	The equation is equivalent to $\frac{dy}{dx} + 2y = e^{-x}$  We have $P=2, Q=e^{-2x}$ General solution: $y = e^{-x} + Ce^{-2x}$ Specific solution: $y = e^{-x}$	0.5 0.5 0.5
9	Area= $\frac{1}{2} OA \times OB  = \frac{1}{2}  (1,2,1) \times (-3,1,0)  = \frac{\sqrt{59}}{3}$	1