Trường ĐH Sư Phạm Sư Phạm Kỹ Thuật Tp.HCM ĐÁP ÁN ĐỀ - MÔN ĐSTT

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Bộ môn Toán

Câu	Ý	Đáp án	Điểm
1		$\overline{A}=\left(\begin{array}{ccc\|c} m & 1 & 1 & 1\\ 1 & m & 1 & 1\\ 1 & 1 & m & 1 \end{array}\right) \xrightarrow{h1\leftrightarrow h3} \left(\begin{array}{ccc\|c} 1 & 1 & m & 1\\ 1 & m & 1 & 1\\ m & 1 & 1 & 1 \end{array}\right) \xrightarrow[h3-m.h1]{h2-h1} \left(\begin{array}{ccc\|c} 1 & 1 & m & 1\\ 0 & m-1 & 1-m & 0\\ 0 & 1-m & 1-m^2 & 1-m \end{array}\right)$ $\xrightarrow{h3+h2} \left(\begin{array}{ccc\|c} 1 & 1 & m & 1\\ 0 & m-1 & 1-m & 0\\ 0 & 0 & (1-m)(m+2) & 1-m \end{array}\right)$	0.75đ
		$m\neq 1\wedge m\neq -2$, $r(A)=r(\overline{A})=n=3$, suy ra hệ có nghiệm duy nhất $\left(\frac{1}{m+2},\frac{1}{m+2},\frac{1}{m+2}\right)$	0.25đ
		$m=-2$, $\overline{A}\sim\left(\begin{array}{ccc\|c} 1 & 1 & -2 & 1\\ 0 & -3 & 3 & 0\\ 0 & 0 & 0 & 3 \end{array}\right)$, $r(A)=r(\overline{A})$, suy ra hệ vô nghiệm	0.25đ
		$m=1$, $\overline{A}\sim\left(\begin{array}{ccc\|c} 1 & 1 & 1 & 1\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 \end{array}\right)$, $r(A)=r(\overline{A})=1<n=3$ Hệ có vô số nghiệm $(1-a-b,a,b),(\forall a,b\in R)$	0.25đ
	Cách 2	$\lvert A\rvert=(m+2)(m-1)^2$ $\lvert A_1\rvert=(m-1)^2;\lvert A_2\rvert=(m-1)^2;\lvert A_3\rvert=(m-1)^2$ + $\lvert A\rvert\neq 0\Leftrightarrow(m+2)(m-1)^2=0\Leftrightarrow m\neq -2\wedge m\neq 1$, hệ có nghiệm duy nhất $\left(\frac{1}{m+2};\frac{1}{m+2};\frac{1}{m+2}\right)$ + $\lvert A\rvert=0\Leftrightarrow(m+2)(m-1)^2=0\Leftrightarrow m=-2\vee m=1$ $m=-2:\lvert A_1\rvert=\lvert A_3\rvert=\lvert A_3\rvert=9\neq 0$, hệ vô nghiệm $m=1:\lvert A_1\rvert=\lvert A_3\rvert=\lvert A_3\rvert=0$, hệ vô số nghiệm $(1-a-b,a,b)$, $a,b\in R$	

2		$A=\begin{pmatrix}-1&3&1&-1\\-1&-1&3&1\\1&1&1&3\\3&-1&1&1\end{pmatrix}\sim\begin{pmatrix}-1&3&1&-1\\0&-4&2&2\\0&4&2&2\\0&8&4&-2\end{pmatrix}\sim\begin{pmatrix}-1&3&1&-1\\0&-4&2&2\\0&0&4&4\\0&0&8&2\end{pmatrix}\sim\begin{pmatrix}-1&3&1&-1\\0&-4&2&2\\0&0&4&4\\0&0&0&-6\end{pmatrix}$	0.5đ
		$A=LU=\begin{pmatrix}1&0&0&0\\1&1&0&0\\-1&-1&1&0\\-3&-2&2&1\end{pmatrix}.\begin{pmatrix}-1&3&1&-1\\0&-4&2&2\\0&0&4&4\\0&0&0&-6\end{pmatrix}$	0.5đ
		$Ax=b\Leftrightarrow LUx=b\Leftrightarrow\begin{cases}LY=b\\Ux=Y\end{cases}$ $LY=b\Leftrightarrow\begin{pmatrix}1&0&0&0\\1&1&0&0\\-1&-1&1&0\\-3&-2&2&1\end{pmatrix}\begin{pmatrix}y_1\\y_2\\y_3\\y_4\end{pmatrix}=\begin{pmatrix}3\\2\\1\\0\end{pmatrix}\Leftrightarrow\begin{pmatrix}y_1\\y_2\\y_3\\y_4\end{pmatrix}=\begin{pmatrix}3\\-1\\3\\1\end{pmatrix}$	0.25 đ
		$Ux=y\Leftrightarrow\begin{pmatrix}-1&3&1&-1\\0&-4&2&2\\0&0&4&4\\0&0&0&-6\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix}=\begin{pmatrix}3\\-1\\3\\1\end{pmatrix}\Leftrightarrow\begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix}=\begin{pmatrix}-1/24\\5/8\\11/12\\-1/6\end{pmatrix}$	0.25 đ
3		$A=\begin{pmatrix}1&1&-2\\2&3&3\\4&5&-1\end{pmatrix}\sim\begin{pmatrix}1&1&-2\\0&1&7\\0&1&7\end{pmatrix}\sim\begin{pmatrix}1&1&-2\\0&1&7\\0&0&0\end{pmatrix}$	0.5đ
		$KerT=\{x\in R^3/T(x)=0\}=\{x\in R^3/Ax=0\}=\left\{x=\begin{pmatrix}9a\\-7a\\a\end{pmatrix},a\neq0\right\}$ $=\left\{x=a\begin{pmatrix}9\\-7\\1\end{pmatrix},a\neq0\right\}=Span\left\{\begin{pmatrix}9\\-7\\1\end{pmatrix}\right\}$	0.5đ
		Cơ sở của KerT là $\left\{\begin{pmatrix}9\\-7\\1\end{pmatrix}\right\}$ và $\dim KerT=1$	0.5đ
4		$w\in ColA$ vì hệ $Ax=w\Leftrightarrow\begin{pmatrix}4&0&4\\6&4&8\\-8&-2&9\end{pmatrix}.\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}-2\\1\\2\end{pmatrix}$ có $\det(A)=96\neq0$, suy ra hệ nghiệm	0.75đ

		w $\notin NulA$ vì $A.w=\begin{pmatrix}4&0&4\\6&4&8\\-8&-2&9\end{pmatrix}.\begin{pmatrix}-2\\1\\2\end{pmatrix}=\begin{pmatrix}0\\8\\32\end{pmatrix}\neq 0$	0.75đ
5		$b_1=4c_1-c_2,\ b_2=-c_1+c_2+c_3,\ b_3=c_2-2c_3$ $P_{C\leftarrow B}=\begin{pmatrix}4&-1&0\\-1&1&1\\0&1&-2\end{pmatrix}$	1đ
		$x=3b_1+4b_2+b_3\Rightarrow [x]_B=\begin{pmatrix}3\\4\\1\end{pmatrix}$ $[x]_C=P_{C\leftarrow B}.[x]_B=\begin{pmatrix}4&-1&0\\-1&1&1\\0&1&-2\end{pmatrix}\begin{pmatrix}3\\4\\1\end{pmatrix}=\begin{pmatrix}8\\2\\2\end{pmatrix}$	1đ
6		$\det(A-\lambda I)=0\Leftrightarrow\begin{vmatrix}5-\lambda&-3&1\\-3&5-\lambda&1\\0&0&8-\lambda\end{vmatrix}=0\Leftrightarrow(8-\lambda)^2(2-\lambda)=0\Leftrightarrow\lambda_1=8;\lambda_2=2$	0.5 đ
		$\lambda_1=8$, giải hệ $(A-8I)X=0\Leftrightarrow\begin{pmatrix}-3&-3&1\\-3&-3&1\\0&0&0\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$ VTR: $u_1=\begin{pmatrix}1\\0\\3\end{pmatrix},u_2=\begin{pmatrix}0\\1\\3\end{pmatrix}$	0.5 đ
		$\lambda_1=2$, giải hệ $(A-2I)X=0\Leftrightarrow\begin{pmatrix}3&-3&1\\-3&3&1\\0&0&6\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$ VTR: $u_3=\begin{pmatrix}1\\1\\0\end{pmatrix}$	0.5đ
		Ma trận khả nghịch $P=\begin{pmatrix}1&0&1\\0&1&1\\3&3&0\end{pmatrix};P^{-1}AP=\begin{pmatrix}8&0&0\\0&8&0\\0&0&2\end{pmatrix}$	0.25đ

		$A^{2023} = PD^{2023}P^{-1} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 3 & 3 & 0 \end{pmatrix}.\begin{pmatrix} 8^{2023} & 0 & 0 \\ 0 & 8^{2023} & 0 \\ 0 & 0 & 2^{2023} \end{pmatrix}.\begin{pmatrix} 1/2 & -1/2 & 1/6 \\ -1/2 & 1/2 & 1/6 \\ 1/2 & 1/2 & -1/6 \end{pmatrix}$ $= \begin{pmatrix} 8^{2023} & 0 & 2^{2023} \\ 0 & 8^{2023} & 2^{2023} \\ 3.8^{2023} & 3.8^{2023} & 0 \end{pmatrix}\begin{pmatrix} 1/2 & -1/2 & 1/6 \\ -1/2 & 1/2 & 1/6 \\ 1/2 & 1/2 & -1/6 \end{pmatrix}$ $= \begin{pmatrix} \frac{1}{2}\left(8^{2023}+2^{2023}\right) & \frac{1}{2}\left(-8^{2023}+2^{2023}\right) & \frac{1}{6}\left(8^{2023}-2^{2023}\right) \\ \frac{1}{2}\left(-8^{2023}+2^{2023}\right) & \frac{1}{2}\left(8^{2023}+2^{2023}\right) & \frac{1}{6}\left(8^{2023}-2^{2023}\right) \\ 0 & 0 & 8^{2023} \end{pmatrix}$	0.25đ
Tổng điểm			10 đ